Fen-Edebiyat Fakültesi	MIDETERM EXAM I	
Name, Surname:	Department:	GRADE
Student No:	Course: Diff Equs	
Signature:	Exam Date: 7/11/2018	

Each problem is worth 25 points. Duration is 90 minutes.

1. Solve the equation $\left(\frac{y}{x}+6x\right)dx+(\ln x-2)\,dy=0,\ x>0.$

Solution: Let $M=\frac{y}{x}+6x$, $N=\ln x-2$. $\frac{\partial M}{\partial y}=\frac{1}{x}=\frac{\partial N}{\partial x}$ so the equation is exact.

$$\frac{\partial \phi}{\partial x}=M \implies \phi=\int\left(\frac{y}{x}+6x\right)dx=y\ln x+3x^2+h(y).$$

$$\frac{\partial \phi}{\partial y}=N \implies \ln x+h'(y)=\ln x-2 \implies h(y)=-2y$$

The solution is

$$y\ln x+3x^2-2y=c \implies y=\frac{c-3x^2}{\ln x-2}$$

2. Given that $y_1(t)=t^2$ is a solution of $t^2y''-4ty'+6y=0,$ $t>0$, use the method of reduction of order to find a second solution.

Solution: Plug $y=vt^2$ into the equation to get $v''t^4=0$, that is $v''=0$, that is $v=c_1t+c_2$, or $y=c_1t^3+c_2t^2$. The second solution can be taken as $y_2=t^3$.

3. Find the solution of the initial value problem $y''+2y'+5y=0,$, $y(0)=0$, $y'(0)=2$. Plot the solution in the $y-t$ plane.

Solution: $\Delta=4-20=-16$ so the roots of the characteristic equation are $r_{1,2}=-1\pm 2i$. The general solution is $y=e^{-t}(c_1\cos 2t+c_2\sin 2t)$. Using the initial condition, we get $y=e^{-t}\sin 2t$. The graph of the function passes from $(0,0)$, is increasing at $(0,0)$, oscillates and decays to the t-axis.

4. Solve the initial value problem $\frac{dy}{dx}=\frac{x^2+y^2}{xy}$, $x>0$, $y(1)=2$. (Hint use the change of variables: $v=y/x$.)

Solution:

$$y=vx \implies \frac{dy}{dx}=x\frac{dv}{dx}+v=\frac{x}{y}+\frac{y}{x}=\frac{1}{v}+v$$

Hence

$$x\frac{dv}{dx}=\frac{1}{v} \implies v\,dv=dx/x \implies \frac{v^2}{2}=\ln x+c \implies (y/x)^2=2(\ln x+c)$$

Using the initial condition, we get $c=2$. The solution is $y^2=2x^2(\ln x+2)$